A Hybrid Image Reconstruction Approach for Ultra-Wide Band Microwave Tomography Featuring Radar Based and Iterative Methods

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***Abstract*— This paper presents a hybrid approach that combines two distinct classes of solutions for microwave tomography; namely radar and tomographic approaches. In this case, the fast computational advantage of radar based methods are utilized in obtaining a more suitable initial guess that can be used as an input to the iterative tomographic process, thus yielding an accurate quantitative result that is less sensitive to the inherited nonlinearity of the inverse scattering problem. Experimentation on numerical results suggest the effectiveness of this approach especially with common imaging setups.**

Keywords—Microwave tomography; Ultra-wide band; conjugate gadient ; inverse problem.

I. INTRODUCTION

There has been a growing interest in microwave imaging techniques in many imaging applications due to the advantages microwaves offer. As in other imaging modalities, microwave imaging relies on illuminating targets under investigation using microwave sources, resulting in scattered information that is collected at several sensing points. The identification of the target requires solving the inverse scattering problem that is well understood to be ill posed, and non-linear in nature [1].

Early attempts to solve this problem, such as the Born or Raytov approximations, involved making linear approximations of the non-linear problem, such as the weak scattering assumption. A more recent comeback of this class of methods were proposed in the form of beam-forming methods such as the Delay and Sum (DAS) method, and its variations, which were specifically proposed for medical imaging (early stage breast cancer detection). The appealing features of the linear approaches lie in its simplicity and computational efficiency, where the solution is obtained in a single step rather than in an iterative procedure. The consequences of these linearized solutions is that these approaches can only produce qualitative images as the field data is lost and replaced by a map of intensity variance, in addition to restrictions of the linear approximation assumption.

In comparison with linear approaches, non-linear algorithms are more accurate and robust making them more appropriate for practical applications, but at a higher complexity and computational cost. Non-linear effects are more pronounced as the illumination frequency increases attempting to achieve higher image resolution [2]. The complexity of the domain under investigation, such as strong or multiple scatterers also contribute to the nonlinearity. To resolve such issues, the problem is formulized as an optimization problem that involves minimizing an error cost function defined as the least squares between measured and calculated data. Among the various approaches, the conjugate gradient method is among the powerful techniques used. However, and as with local minimization approaches, the problem of local minima and sensitivity to the initial guess are usually encountered. In this work, the hybrid approach will involve combining the DAS method and the Polak-Ribière conjugate gradient algorithm.

II. MATHMATICAL MODEL

The proposed hybrid procedure combines the DAS method with the tomographic-optimization procedure in the form of the conjugate gradient minimization. The Delay and Sum beam-forming algorithm, introduced by Hagness in the late 1990's [3] is based on confocal microwave imaging and mono-static measurements. In this arrangement, the antenna element is used to illuminate the target and record the reflected energy, repeating the process for each antenna in turn. The reconstruction algorithm involves time-shifting, and summing the signals scattered from the target to create a synthetic focal point. An energy profile is created by varying the position of the synthetic focus within the imaging domain, mathematically:

$$I(r) = \int_0^{T_{win}} \left[\sum_{n=1}^{M} S_n\big(t - \tau_n(r)\big) \right]^2 dt \qquad (1)$$

Where M denotes the number of antennas, S_n denotes the n^{th} backscattered signal. The time delay $\tau_n(r)$ is given as:

$$\tau_n(r) = 2\frac{|r - r_n|}{vT_s} \qquad (2)$$

where $|r - r_n|$ is the distance between the transmitting antenna r_n, and the focal point r, v is the propagation speed, T_{win}, and T_s are the window length and sampling interval respectively.

In this work, we follow the multi-static measurements approach where measurements are obtained by illuminating the domain under investigation by each displaced transmitter, while the remaining receivers record the backscattered signals. This approach is fully described in [4].

Tomography methods require solutions to the forward and the inverse scattering problems. The forward problem calculates the scattered fields as a function of geometry and electrical properties. In contrast the inverse problem seeks to find the permittivity distribution utilizing the scattered signal measurements. The problem is usually expressed as unconstrained optimization, from which the solutions are developed by minimizing the non-linear cost function. The optimization procedures used here are described in our earlier publications [5]. A summarized version is presented here for the sake of thoroughness. The goal of optimization process is to solve the material properties $\boldsymbol{x}=(\varepsilon)$ of the medium over the domain Ω, which minimizes the cost function $F(x)$ defined as the least square between the calculated field E^{calc} and measured field E^{meas}. Assuming a total of M transmitters and N receivers, the residual error of the N receiving locations for the m^{th} projection over the time t in TM mode is define as:

$$r(\boldsymbol{x}) = \frac{1}{2}\sum_{n=1}^{N} \int_T \int_\Omega \left(E_z^{calc}{}_n(\boldsymbol{x}) - E_z^{meas}{}_n\right)^2 d\Omega dt\delta \quad (3)$$

where δ is the dirac delta function. Subsequently, the cost function for the total M projections is:

$$F(\boldsymbol{x}) = \sum_{m=1}^{M} r_m(\boldsymbol{x}) \qquad (4)$$

where m=1,2,..,M, is the transmitter locations.

For a multivariable vector $\boldsymbol{x}$, the gradient $\boldsymbol{g} = \nabla F(\boldsymbol{x})$ points to the direction of the maximum rate of increase of F at $\boldsymbol{x}$. Consequently, the direction of the negative gradient points to the minimizer, and which sets the search direction in our case.

To calculate the gradient, we follow the adjoint state method for inverse scattering problems which requires introducing Maxwell's equation as constraints to the functional by using Lagrange multipliers. This results in the formulation of the adjoint problem that is solved by backward time propagation of the residual errors, and which can be evaluated using the FDTD in the same fashion as the direct problem. Following the derivation, the gradient of the cost function F with respect to permittivity of the i^{th} & j^{th} unknown pixels is given as:

$$\nabla F(\boldsymbol{x}) = \sum_{m=1}^{M}\left(\sum_{nt=1}^{NT-1}\sum_{i,j\,\epsilon\Omega}^{Ni\ Nj} e_z \frac{dE_z}{dt}\Delta x\Delta y\, \Delta t\right) \quad (5)$$

An initial guess is required to start the iterative process. Usually, this guess is selected to be the homogeneous medium of the imaging domain under investigation. The iterative process is sensitive to the proximity of the initial guess to the actual solution and plays a significant factor in the convergence process. Here, solution obtained via DAS algorithm is processed, and used as the initial guess of the iterative problem. To achieve this, the DAS solution is processed in two steps which require : i) increasing the energy intensity discrepancy to remove possible artifacts resulting from DAS reconstruction, and ii) remap the energy levels to appropriate field value data (electrical permittivity) according to the priori information in the setup.

The first step is accomplished by intensifying the energy at the focal point using the power parameter n (Eqn. 6). This constant is not arbitrary, and may differ from one imaging case to the other depending on the target configuration as it will be demonstrated in the next section. On the other hand, the remapping of the energy levels into the permittivity map (i^{th}, and j^{th} pixels) is accomplished by linearly expanding the 0-100% of the normalized energy level span into the range that corresponds to the minimum permittivity ε_{rmin} and maximum permittivity $\varepsilon_{r\,max}$values known from priori information such that:

$$x(i,j) = \frac{d\varepsilon_r}{dI} I^n(i,j) + \varepsilon_{r\,min} \qquad (6)$$

and, $$\frac{d\varepsilon_r}{dI} = \frac{\varepsilon_{r\,max} - \varepsilon_{r\,min}}{1} \qquad (7)$$

where $\frac{d\varepsilon_r}{dI}$ is the constant found from the slope.

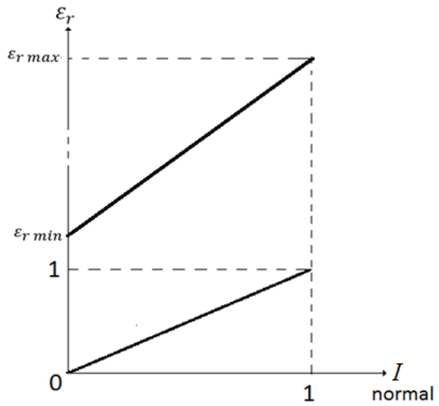
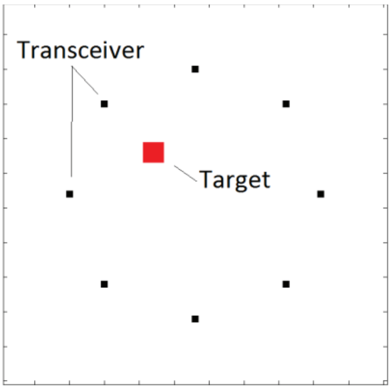


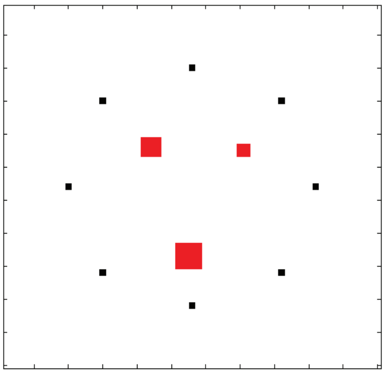
Figure 1: linear mapping of Energy levels to relative permittivity values

III. NUMERICAL RESULTS

To investigate the method proposer earlier, a numerical model is set using the FDTD method. The forward setup follows that described in [6]. In this case however, a free space homogeneous region is assumed with the presence of multiple sized square targets having and edge length of 10.5 mm, 8 mm, and 5 mm (Fig. 2). The targets are assumed to have a relative permittivity values of 3 and 5 as demonstrated for each test case. A total of 8 transceivers are uniformly distributed around the imaging domain with a 0-5 GHz impulse used to illuminate the domain. Two test cases are created where in the first; the medium sized target is used only, while in the second, all targets are used with actual locations as described in Figure 2. Data recordings for both cases are obtained by sequentially exciting the transceivers to illuminate the targets via the FDTD forward solver.



(a)

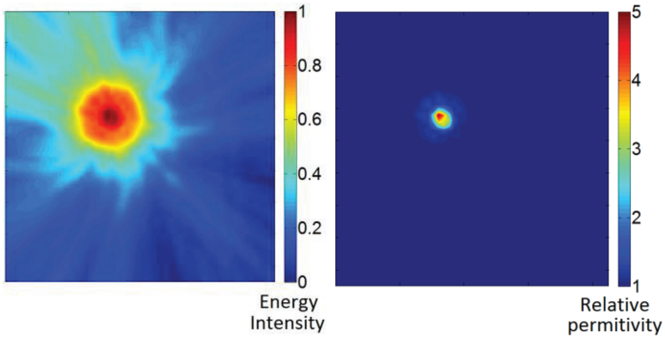


(b)

Figure 2: Actual location of test targets; (a) one target, 8 mm, and (b) three targets, 5mm, 8 mm, and 10.5 mm

A. *Test Case of single scatterer*

Following the procedure described earlier, a medium size single scatterer target having a relative permittivity of 5 is assumed. As a start, the DAS method is used to reconstruct the initial image. Figure 3(a) shows the result of the DAS reconstruction, where the qualitative nature of the image can be clearly observed from the intensity map. To obtain the qualitative result required for the iterative process in this case, the minimum and maximum permittivities priorly known ($\varepsilon_{r\,min} = 1, \varepsilon_{r\,max} = 5$) are used in Equation (6). To obtain the best value of *n*, a single line minima search is conducted relative to the error functional defined in Equation (3). Figure 4 illustrates a plot of the resulting error due to a sweep of *n* values. With reference to this figure, a value of *n*=14 yields the lowest error, and thus the most appropriate to use. Figure 3(b) shows the quantitative image obtained by direct application of Equation (3). As it can be seen from the figure, the variance in intensity has been replaced with variance in permittivity levels. In addition, the direct examination of Figure 3(b) and 2(a) reveals that the target has been correctly identified, although features of the target geometry have been lost.



(a) (b)

Figure 3: DAS reconstruction showing (a) qualitative image, and (b) quantitative converted image.

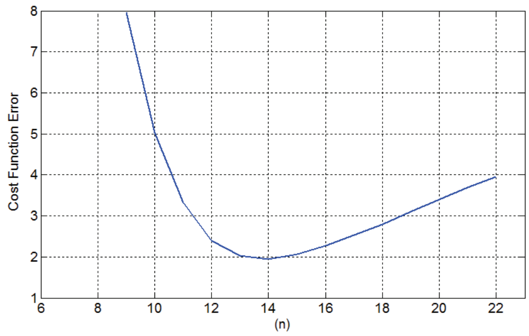
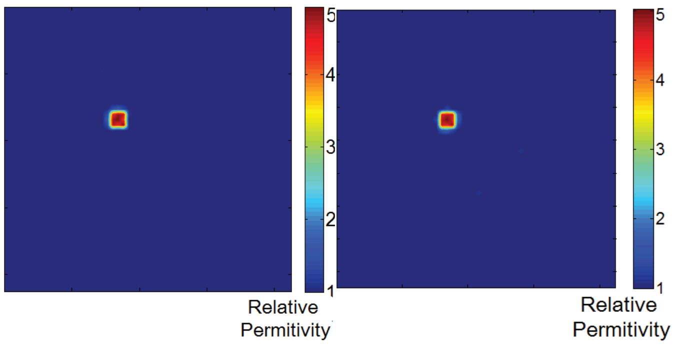


Figure 4: Cost functional error for different values of the power parameter *n*.

To evaluate the effectiveness the suggested approach, two iterative reconstruction were performed. The first incorporated the result obtained from the quantitative derived DAS output (Fig. 3(b)) as the initial starting point, while a blank homogenous initial guess was chosen for the second. The iterative process is terminated whenever the error difference between two consecutive iterations is less than 5%. Figures 5, and 6 show the reconstructed image of both approaches and the corresponding convergence trend. Examining Fig.5(b), and by comparison with Fig. 3(b), it can be observed that the geometrical features lost during the DAS reconstruction have been restored. Both iterative results shown in Fig 5(a) and 5(b) show that the target has been correctly identified in terms of size, location, permittivity, and geometrical features. However, the suggested hybrid approach requires half the number of iterations, indicating about 50% reduction in reconstruction time.



(a) (b)

Figure 5: Iterative reconstruction showing: (a) with the hybrid derived (converted) DAS as initial guess, and (b) with a background homogeneous data set.

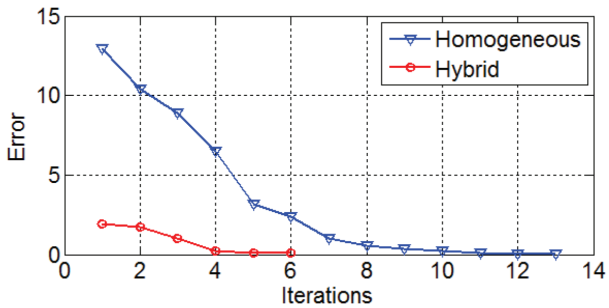
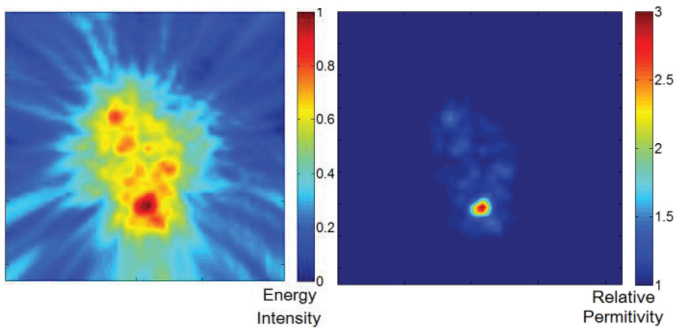


Figure 6: Iterative convergence trend for the quantitative derived (converted) DAS as initial guess, and with a background homogeneous data set.

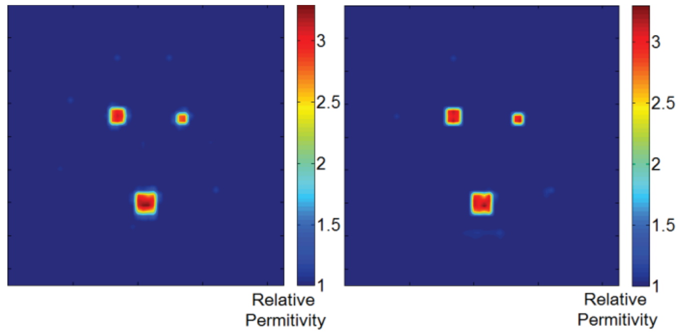
B. Test Case of multiple scatterers

In this case, three square targets of varying sizes are assumed inside the imaging domain. The targets are assumed to have a relative permittivity of 3, while processing steps identical to those performed for the single scatterer target are repeated. First, the modified DAS reconstruction is obtained where the value of n corresponding to the lowest error was found to be n=7 in this case (Fig. 7). Similar to the case of the single scatterer, two iterative reconstructions are performed starting with both converted DAS output and homogenous solution as the initial first step of the iterative process. Fig. 8 and 9 demonstrate the reconstructed images and their corresponding convergence trend.



(a) (b)

Figure 7: DAS reconstruction for multiple targets showing (a) qualitative image, and (b) quantitative converted image.



(a) (b)

Figure 8: Iterative reconstruction showing: (a) with the hybrid quantitative derived (converted) DAS as initial guess, and (b) with a background homogeneous data set.

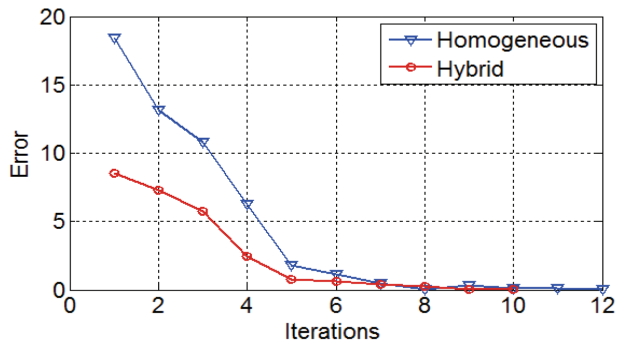


Figure 9: Iterative convergence trend for the quantitative derived (converted) DAS as initial guess, and with a background homogeneous data set.

The examination of the convergence trend reveals that the hybrid approach required more iteration in the case of multiple scatterers when compared to the single target. This problem is expected as quantitative methods usually suffer with increasing domain complexity, thus losing the advantage as to when compared to simple setups [7]. This trend is expected to increase with the increasing nonlinear factors that influence the scattered signals.

IV. CONCLUSION

In this paper, a hybrid image reconstruction approach is introduced. This approach combines qualitative methods in the form of the DAS beam forming method and utilizing the end result as the initial guess to start the iterative process after proper processing. The numerical results presented indicate that the convergence trend, and thus, the reconstruction time is faster when compared to the conventional iterative process. However, and as the imaging setup becomes more complex, this advantage is reduced. Thus, the quality of the image obtained via DAS becomes a major determinant to the convergence, as well as the final image quality.

In summary, the suggested approach is more suitable for applications that usually encounter single scatterers, as in the case of breast cancer tomography. The incorporation of multiple targets, vast permittivity contrast, or factors that increase the nonlinearity of the inverse problem in general are expected to impose their limitations. The experimental validation as well as the investigation of other alternative linear techniques will be the focus of future research.

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